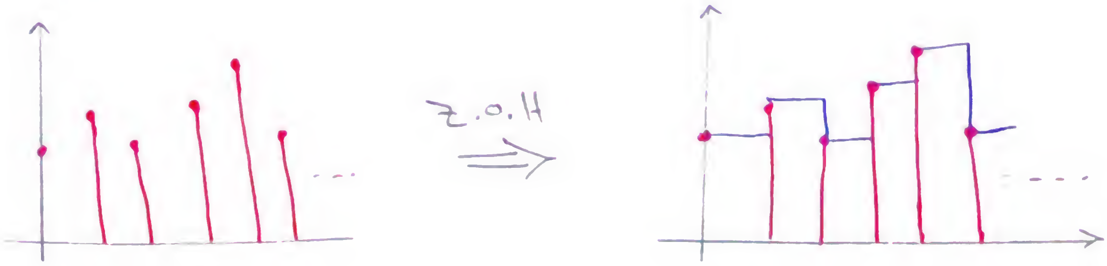
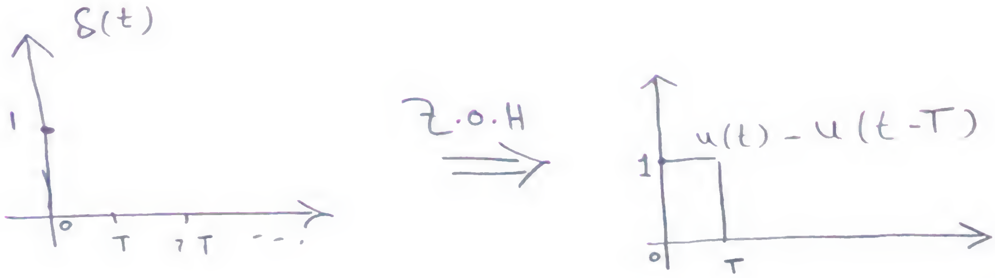
Digital

22/2/2016 الاثنين د. عرفة محاضرة [2]

The T.F. of Z.O.H :-

Z.O.H = Zero order hold





$$\text{T.F. for Z.O.H} = G_{Z.O.H} = G_h(S) = \frac{O/P(S)}{I/P(S)}$$

Laplace Transform

$$= \frac{L\,(u(t) - u(t-T))}{L\,(d(t))}$$

$$= \frac{1/s - \frac{e^{-Ts}}{s}}{1} = \frac{1}{s}\left(1 - e^{-Ts}\right)$$

$$G_{Z.O.H}(S) = \frac{1 - e^{-Ts}}{S}$$

\# Z.T $G_h(S) \cdot G_1(S)$

$$= Z\left[\frac{1 - e^{-TS}}{S}\, G_1(S)\right]$$

رجعناها لأصلها في T.domain
وهنا ت Z.T

$$= (1 - Z^{-1})\; Z\left[\frac{G_1(S)}{S}\right]$$

Z.T.

* The Z.O.H gain = 1

$$Z.\left[G_h(s)\right] = Z\left[\frac{1-e^{-Ts}}{s}\right]$$

$$= (1-z^{-1})\; Z\left[\frac{1}{s}\right]$$

$$= \left(\frac{z-1}{z}\right)\cdot\left(\frac{z}{z-1}\right) = 1$$

Pulse T.F.

The discrete or digital T.F. is called pulse T.F.

$$\text{pulse T.F.} = \frac{C^*(s)}{R^*(s)} = \frac{C(z)}{R(z)}$$

① Pulse T.F. from difference equations

for ex:

$$y(k-1) + 2y(k) + 3y(k-2) = r(k)$$

Find the impulse T.F. $= \dfrac{Y(z)}{R(z)}$

$\Downarrow$ ZT

$$z^{-1}(z) + 2Y(z) + 3z^{-2}Y(z) = R(z)$$

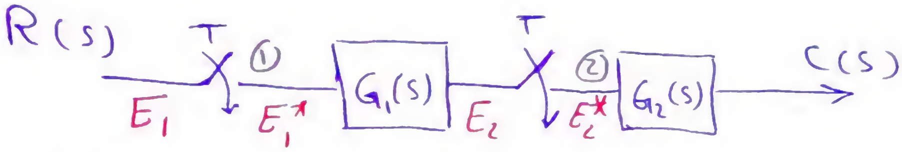
$$(3z^{-2}+z^{-1}+2)\,Y(z) = R(z)$$

$$\Longrightarrow \frac{Y(z)}{R(z)} = \frac{1}{3z^{-2}+z^{-1}+2}$$

$$\frac{Y(z)}{R(z)} = \frac{z^2}{2z^2+z+3}$$

[2] Pulse T.F from block diagram

① open loop System



$C(s) = G_2(s)\, E_2^*$ ① $\xrightarrow{\text{Staring}}$ $C^*(s) = G_2^*(s)\, E_2^*$ ①*

$E_2(s) = R^*\, G_1(s)$; $R^* = E_1^*$ $\xrightarrow{\text{Staring}}$ $E_2^*(s)\, G_1^*(s)\, R^*$ ②*

from ②* in ①*

$$C^*(s) = G_1^*(s) \,.\, G_2^*(s) \,.\, R^*$$

$$\frac{C^*(s)}{R^*(s)} = G_1^*(s) \,.\, G_2^*(s)$$

if we remove the sampler ②

$$C(s) = G_1(s) \,.\, G_2(s)\, R^*(s)$$

Staring $\Downarrow$

$$C^*(s) = \overline{G_1\, G_2}^*(s) \quad R^*(s)$$

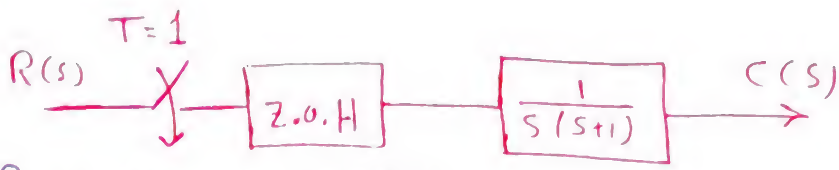
we deal with G_1, G_2 as one continuous signal

So $\overline{G_1\, G_2}^*(s)$ means staring of $\overline{G_1\, G_2(s)}$ (one signal)

$$\frac{C^*(s)}{R^*(s)} = \overline{G_1\, G_2\, (s)}^*$$

$$\frac{C(z)}{R(z)} = \overline{G_1\, G_2}(z)$$

Ex:



① Find $T.F. = \dfrac{C(z)}{R(z)}$

② Find the unit step response

$$\frac{C(z)}{R(z)} = \overline{G_1 G_2(z)}$$

$$= Z\left[\frac{1-e^{-Ts}}{s^2(s+1)}\right]$$

$$= (1-z^{-1})\; Z\left[\frac{1}{s^2(s+1)}\right]$$

$$= \left(\frac{z-1}{z}\right) \cdot Z\left[\frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s+1}\right]$$

$A_1 = 1$, $A_3 = 1$, $A_2 \,\|\, s=1 \Rightarrow \frac{1}{2} = A_1 + A_2 + A_3/2$

$\Rightarrow A_2 = -1$

$$= \frac{z-1}{z}\; Z\left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right]$$

$t \qquad 1 \qquad e^{-t}$

$$= \left(\frac{z-1}{z}\right)\left[\frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-T}}\right]$$

$$= \left(\frac{z-1}{\cancel{z}}\right)\cancel{z}\left[\frac{1}{(z-1)^2} - \frac{1}{z-1} + \frac{1}{z-e^{-1}}\right] \qquad \Big|\; e^{-1} = 0.368$$

$$= \frac{z-0.368-(z-1)(z-0.368)+(z-1)^2}{(z-1)(z-0.368)}$$

$$\frac{C(z)}{R(z)} = \frac{0.368z + 0.264}{(z-1)(z-0.368)} \Rightarrow$$ أول مطلوب

② unit step response $\Rightarrow$ ثاني مطلوب

$r(t) = 1$

$$R(z) = \frac{z}{z-1}$$

$$C(z) = \left[\frac{0.368z + 0.264}{(z-1)(z-0.368)}\right] \cdot R(z)$$

$$C(z) = \left[\quad\right] \frac{z}{z-1}$$

$$C(z) = \frac{(0.368z + 0.264)z}{(z-1)^2(z-0.368)} \Downarrow z^{-1}.T$$

$$C(z) = z\left[\frac{0.368z + 0.264}{(z-1)^2(z-0.368)}\right]$$

$$= z\left[\frac{A_1}{(z-1)^2} + \frac{A_2}{(z-1)} + \frac{A_3}{(z-0.368)}\right]$$

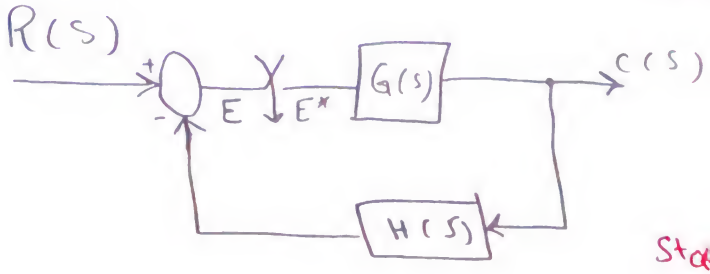
$A_1 = 1$
$A_2 = -1$
$A_3 = 1$

$$C(z) = \frac{z}{(z-1)^2} - \frac{z}{(z-1)} + \frac{z}{(z-0.368)} \Downarrow z^{-1}T$$

$$\boxed{C(K) = K - u(K) + (0.368)^K}$$

$\Leftarrow$ Unit step response

② close loop systems



① اعمل تسمية لدخل وخرج ال Sampler الموجود
E_1 , E_2 , E_3 . . .

② هات معادلة الخرج الرئيسي بالإضافة لمعادلات دخل ال Sampler

③ لو فيه تعويضات اعملها قبل ال Staring

④ اعمل Staring

$$\boxed{C(s) = G(s)\, E^*(s)} \quad — ①$$

$$E(s) = R(s) - H(s) . C(s)$$

$$\boxed{E(s) = R(s) - GH(s)\, E^*(s)} \quad — ②$$

$$C^*(s) = G^*(s)\, E^*(s) \quad \text{"Staring of 1"} \quad ①^*$$

$$E^*(s) = R^*(s) - \overline{GH(s)}^*\, E^*(s) \quad \text{"Staring of 2"} \quad ②^*$$

$$\left(1 + \overline{GH(s)}^*\right) E^*(s) = R(s)$$

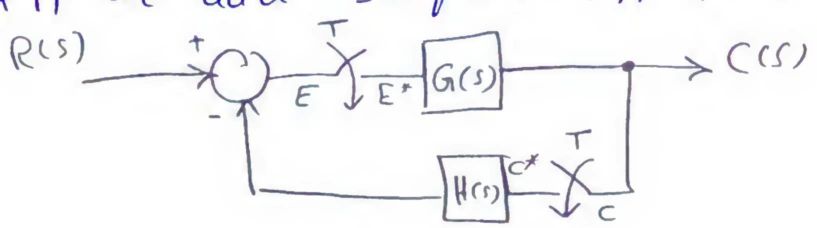
$$E^*(s) = \frac{R^*(s)}{1 + \overline{GH(s)}^*} \quad ③$$

from ③ in $①^*$

$$C^*(s) = \frac{G^*(s)\, R^*(s)}{}$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + \overline{GH(s)}^*} \Longrightarrow \boxed{\frac{C(z)}{R(z)} = \frac{G(z)}{1 + GH(z)}}$$

* if we add sampler in feed back



$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + G(z)H(z)}$$

* Proof

$C(s) = G(s)\,E^*(s)$ ① $\xrightarrow{\text{Staring}}$ $C^*(s) = G^*(s)\,E^*(s)$

$E = R - H(s).\,C^*(s)$ ② $\xrightarrow{\text{Staring}}$ $E^*(s) = R^* - H^*.C^*(s)$

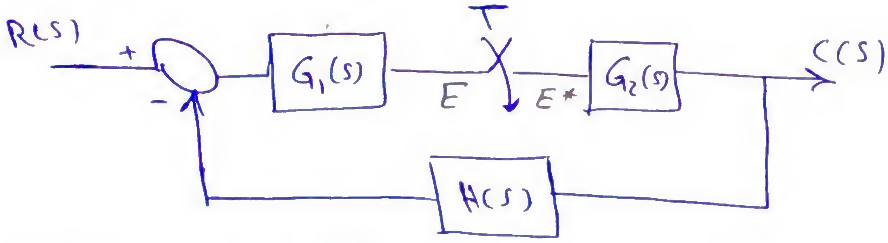
$E^*(s) = R^* - H^*(s).\,G^*(s)E^*(s)$

$\Rightarrow \left[1 + G^*(s)\,H^*(s)\right] E^*(s) = R^*(s)$

$$E^*(s) = \frac{R^*(s)}{1 + G^*(s).\,H^*(s)} \quad ③$$

From ③ in ① $\Rightarrow$ $$\frac{C^*}{R^*} = \frac{G^*(s)}{1 + G^*(s).\,H^*(s)}$$

* Ex 3



$C(s) = G_2(s)\,E^*(s) \Longrightarrow C^*(s) = G_2^*(s)\,D^*(s)$

$E^*(s) = G_1\left[R(s) - G_2(s)H(s)E^*(s)\right]$

$= G_1R(s) - G_1G_2H(s)\,E^*(s)$ ②

$\Longrightarrow E^*(s) = \overline{G_1R(s)}^* - \overline{G_1G_2H(s)}^*\,E^*(s)$

↳ we can't find Pulse T.F.

Response ولكن ممكن نجيب Impulse T.F متعذر نجيب

$(1 + \overline{G_1G_2H(s)}^*)\,E^*(s) = \overline{G_1R}^*(s)$

$$E^*(s) = \frac{\overline{G_1R}^*(s)}{1 + \overline{G_1G_2H(s)}^*}$$

$\Longrightarrow$ Continue

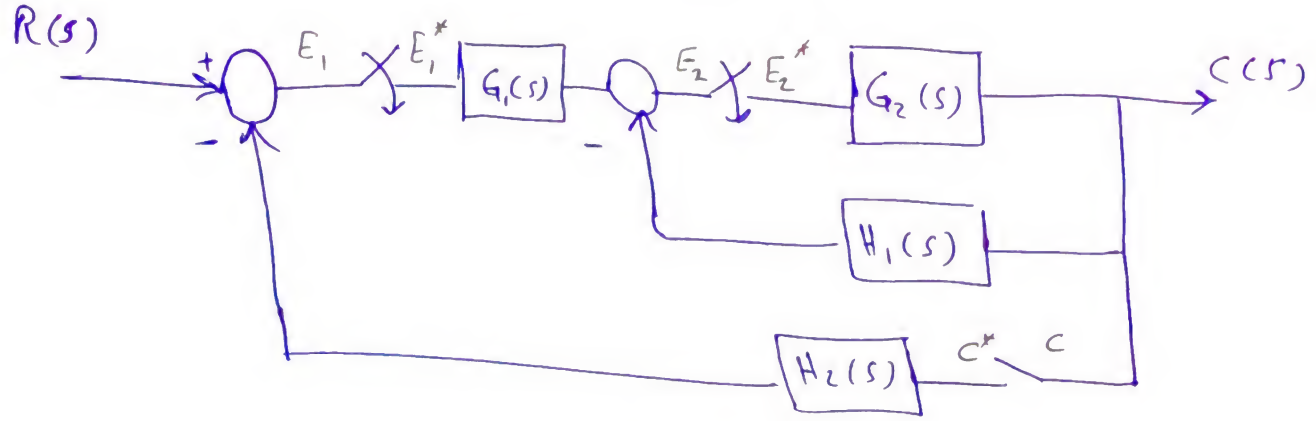
$$C^*(s) = \frac{G_2^* \, \overline{G_1 R}^*(s)}{1 + \overline{G_1 G_2 H}^*(s)}$$

$$C(z) = \frac{G_2(z)\, \overline{G_1 R}(z)}{1 + \overline{G_1 G_2 H}(z)}$$

The System output

Example :-

سؤال امتحان مضمون



Signal flow graph اتعامل زي ال

$$\frac{C(z)}{R(z)} = \frac{G_1(z)\, G_2(z)}{1 + \overline{G_2 H_1}(z) + G_1 G_2 H_2(z)}$$

استعملها للتأكد فقط

$$C(s) = G_2(s)\, E_2^*(s) \longrightarrow ①$$

$$E_2(s) = G_1(s)\, E_1^* - G_2 H_1 E_2^* \longrightarrow ②$$

$$E_1(s) = R(s) - H_2(s)\, C^*(s) \longrightarrow ③$$

$$C^*(s) = G_2^*(s)\, E_2^*(s) \longrightarrow ①^*$$

$$E_2^*(s) = G_1^*(s)\, E_1^*(s) - \overline{G_2 H_1}^*\, E_2^*(s) \longrightarrow ②^*$$

$$E_1^*(s) = R^*(s) - H_2^*(s)\, G_2^*(s).\, E_2^*(s) \longrightarrow ③^*$$

from ③* in ②*

$$E_2^*(s) = G_1^*\left(R^*(s) - H_2^*(s).G_2^*(s)E_2^*(s)\right) - \overline{G_2H_1(s)}^*\,E_2^*(s)$$

$$E_2^* = G_1^*R^* - G_1^*G_2^*H_2^*E_2^*(s) - \overline{G_2H_1(s)}^*E_2^*(s)$$

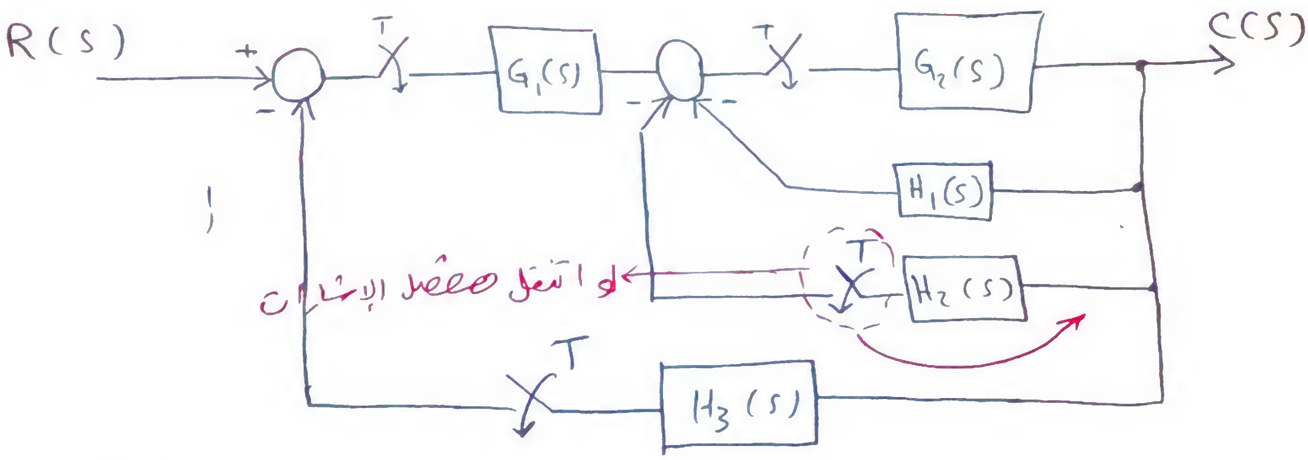
$$\left(1 + \overline{G_2H_1}^*(s) + G_1^*(s).G_2^*(s).H_2^*(s)\right)E_2^*(s) = G_1^*(s)R^*(s)$$

$$\Rightarrow E_2^*(s) = \frac{G_1^*(s).R^*(s)}{1 + \overline{G_2H_1(s)}^* + G_1^*(s).G_2^*(s).H_2^*(s)} \Rightarrow ④$$

from ④ in ①*

$$\frac{C^*(s)}{R^*(s)} = \frac{G_1^*(s)\;G_2^*(s)}{1 + \overline{G_2H_1}^*(s) + G_1^*(s).G_2^*(s).H_2^*(s)}$$

$$\Rightarrow \frac{C(z)}{R(z)} = \frac{G_1(z).G_2(z)}{1 + \overline{G_2H_1(z)} + G_1(z)\,G_2(z).H_2(z)}$$



$$\frac{C(z)}{R(z)} = \frac{G_1(z)\;G_2(z)}{1 + \overline{G_2H_1(z)} + \overline{G_2H_2(z)} + G_1(z)\,\overline{G_2H_3(z)}}$$